GRE®

GRADUATE RECORD EXAMINATIONS®

Math Review
Large Print (18 point) Edition
Chapter 3: Geometry

The GRE® Math Review consists of 4 chapters: Arithmetic, Algebra, Geometry, and Data Analysis. This is the Large Print edition of the Geometry Chapter of the Math Review. Downloadable versions of large print (PDF) and accessible electronic format (Word) of each of the 4 chapters of the Math Review, as well as a Large Print Figure supplement for each chapter are available from the GRE® website. Other downloadable practice and test familiarization materials in large print and accessible electronic formats are also available. Tactile figure supplements for the 4 chapters of the Math Review, along with additional accessible practice and test familiarization materials in other formats, are available from ETS Disability Services Monday to Friday 8:30 a.m. to 5 p.m. New York time, at 1-609-771-7780, or 1-866-387-8602 (toll free for test takers in the United States, U.S. Territories, and Canada), or via email at stassd@ets.org.

The mathematical content covered in this edition of the Math Review is the same as the content covered in the standard edition of the Math Review. However, there are differences in the presentation of some of the material. These differences are the result of adaptations made for presentation of the material in accessible formats. There are also slight differences between the various accessible formats, also as a result of specific adaptations made for each format.

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Overview of the Math Review

The Math Review consists of 4 chapters: Arithmetic, Algebra, Geometry, and Data Analysis.

Each of the 4 chapters in the Math Review will familiarize you with the mathematical skills and concepts that are important to understand in order to solve problems and reason quantitatively on the Quantitative Reasoning measure of the GRE® revised General Test.

The material in the Math Review includes many definitions, properties, and examples, as well as a set of exercises (with answers) at the end of each chapter. Note, however, that this review is not intended to be all-inclusive—there may be some concepts on the test that are not explicitly presented in this review. If any topics in this review seem especially unfamiliar or are covered too briefly, we encourage you to consult appropriate mathematics texts for a more detailed treatment.

Overview of this Chapter

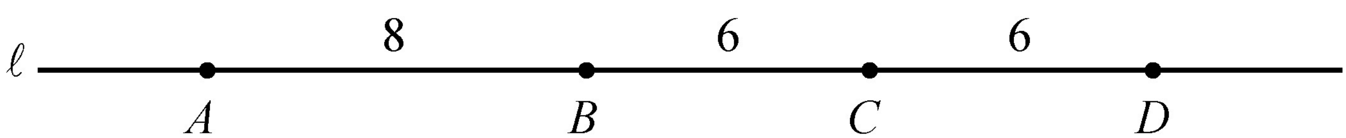
The review of geometry begins with lines and angles and progresses to other plane figures, such as polygons, triangles, quadrilaterals, and circles. The chapter ends with some basic three-dimensional figures. Coordinate geometry is covered in the Algebra chapter.

3.1 Lines and Angles

Plane geometry is devoted primarily to the properties and relations of plane figures, such as angles, triangles, other polygons, and circles. The terms "point," "line," and "plane" are familiar intuitive concepts. A **point** has no size and is the simplest geometric figure. All geometric figures consist of points. A **line** is understood to be a straight line that extends in both directions without end. A **plane** can be thought of as a floor or a tabletop, except that a plane extends in all directions without end and has no thickness.

Given any two points on a line, a **line segment** is the part of the line that contains the two points and all the points between them. The two points are called **endpoints**. Line segments that have equal lengths are called **congruent line segments**. The point that divides a line segment into two congruent line segments is called the **midpoint** of the line segment.

In Geometry Figure 1 below, A, B, C, and D are points on line ℓ.

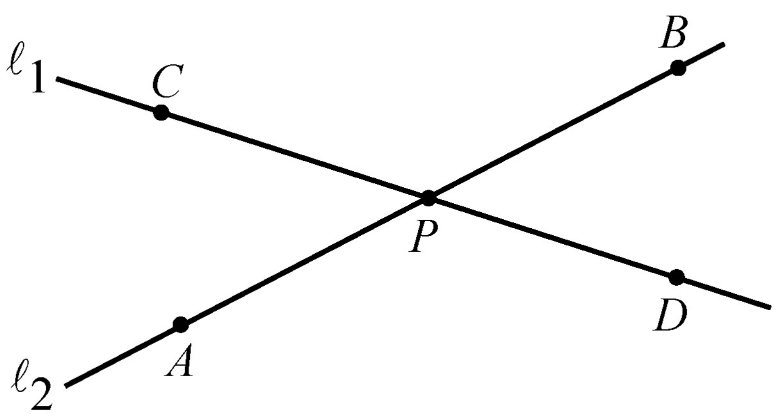


Geometry Figure 1

Line segment AB consists of points A and B and all the points on the line between A and B. According to Geometry Figure 1 above, the lengths of line segments AB, BC, and CD are 8, 6, and 6, respectively. Hence, line segments BC and CD are congruent. Since C is halfway between B and D, point C is the midpoint of line segment BD.

Sometimes the notation AB denotes line segment AB, and sometimes it denotes the **length** of line segment AB. The meaning of the notation can be determined from the context.

When two lines intersect at a point, they form four **angles**, as indicated in Geometry Figure 2 below. Each angle has a **vertex** at point P, which is the point of intersection of the two lines.

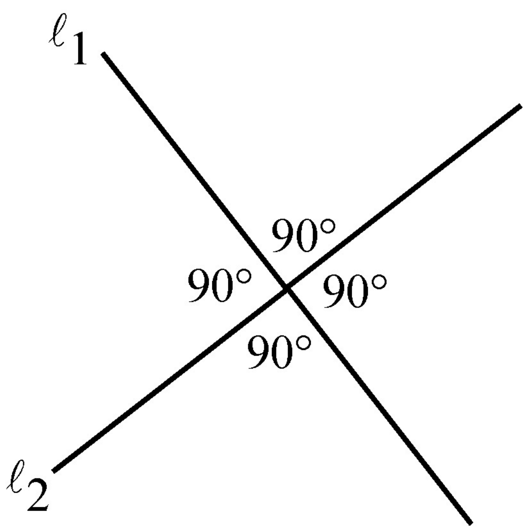


Geometry Figure 2

In Geometry Figure 2, angles APC and BPD are called **opposite angles**, also known as **vertical angles**. Angles APD and CPB are also opposite angles. Opposite angles have equal measures, and angles that have equal measures are called **congruent angles**. Hence, opposite angles are congruent. The sum of the measures of the four angles is 360°.

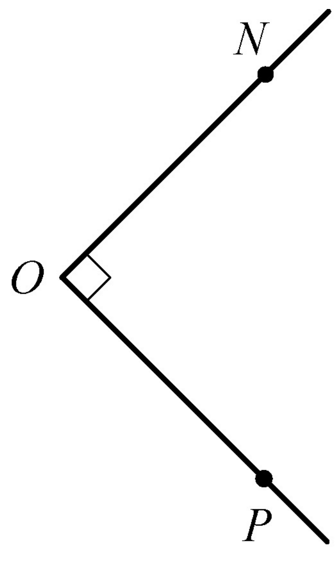
Sometimes the angle symbol $\angle$ is used instead of the word "angle." For example, angle *APC* can be written as $\angle APC$.

Two lines that intersect to form four congruent angles are called **perpendicular lines**. Each of the four angles has a measure of 90°. An angle with a measure of 90° is called a **right angle**. Geometry Figure 3 below shows two lines, ℓ_1 and ℓ_2, that are perpendicular, denoted by $\ell_1 \perp \ell_2$.



Geometry Figure 3

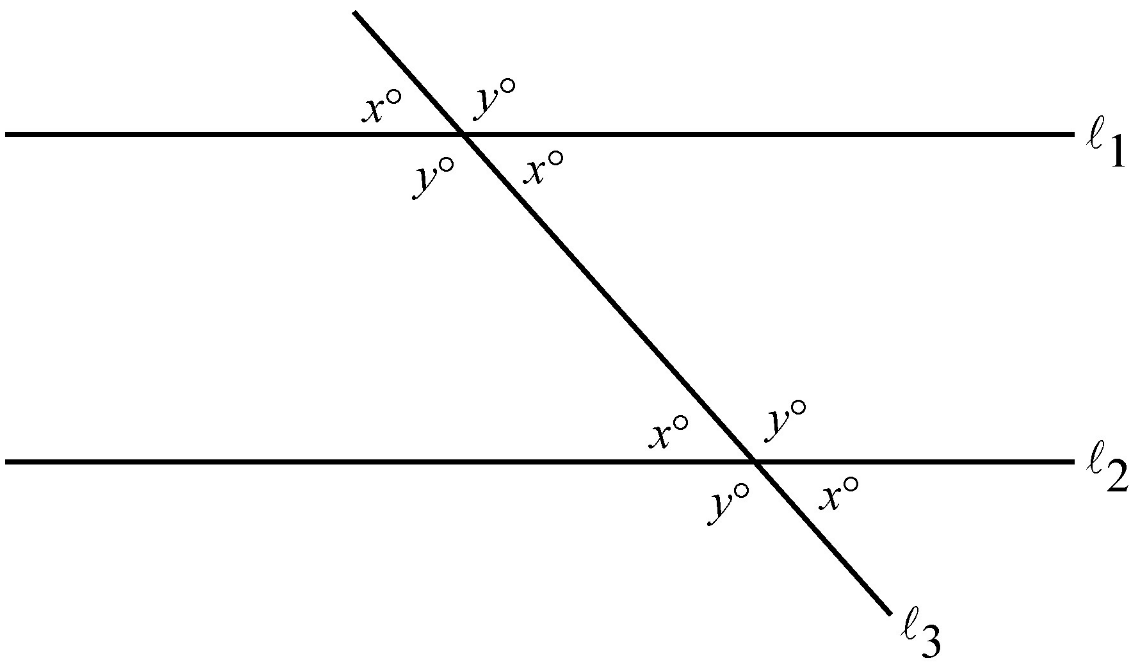
A common way to indicate that an angle is a right angle is to draw a small square at the vertex of the angle, as shown in Geometry Figure 4 below, where *PON* is a right angle.



Geometry Figure 4

An angle with measure less than 90° is called an **acute angle**, and an angle with measure between 90° and 180° is called an **obtuse angle**.

Two lines in the same plane that do not intersect are called **parallel lines**. Geometry Figure 5 below shows two lines, ℓ_1 and ℓ_2, that are parallel, denoted by $\ell_1 \parallel \ell_2$. The two lines are intersected by a third line, ℓ_3, forming eight angles.

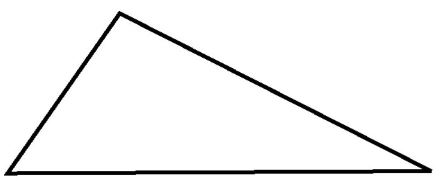


Geometry Figure 5

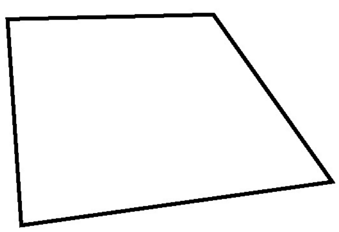
Note that four of the eight angles in Geometry Figure 5 have the measure $x°$, and the remaining four angles have the measure $y°$, where $x + y = 180$.

3.2 Polygons

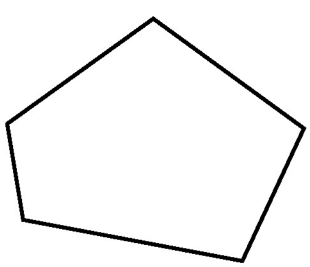
A **polygon** is a closed figure formed by three or more line segments, called **sides**. Each side is joined to two other sides at its endpoints, and the endpoints are called **vertices**. In this discussion, the term “polygon” means “convex polygon,” that is, a polygon in which the measure of each interior angle is less than 180°. Geometry Figure 6 below contains examples of a triangle, a quadrilateral, and a pentagon, all of which are convex.



Triangle
(3 sides)



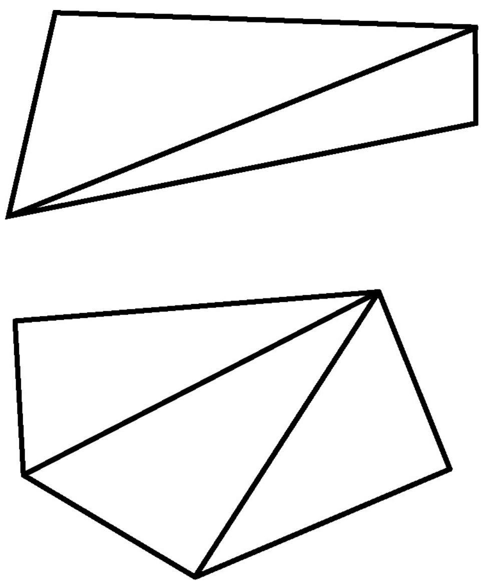
Quadrilateral
(4 sides)



Pentagon
(5 sides)

Geometry Figure 6

The simplest polygon is a **triangle**. Note that a **quadrilateral** can be divided into 2 triangles by drawing a diagonal; and a **pentagon** can be divided into 3 triangles by selecting one of the vertices and drawing 2 line segments connecting that vertex to the two nonadjacent vertices, as shown in Geometry Figure 7 below.



Geometry Figure 7

If a polygon has n sides, it can be divided into $n-2$ triangles. Since the sum of the measures of the interior angles of a triangle is 180°, it follows that the sum of the measures of the interior angles of an n-sided polygon is $(n-2)(180°)$. For example, since a quadrilateral has 4 sides, the sum of the measures of the interior angles for a quadrilateral is $(4 - 2)(180°) = 360°$; and since a **hexagon** has 6 sides, the sum of the measures of the interior angles for a hexagon is $(6 - 2)(180°) = 720°$.

A polygon in which all sides are congruent and all interior angles are congruent is called a **regular polygon**. For example, since an **octagon** has 8 sides, the sum of the measures of the interior angles of an octagon is $(8 - 2)(180°) = 1{,}080°$. Therefore, in a **regular octagon** the measure of each angle is $\frac{1{,}080°}{8} = 135°$.

The **perimeter** of a polygon is the sum of the lengths of its sides. The **area** of a polygon refers to the area of the region enclosed by the polygon.

In the next two sections, we will look at some basic properties of triangles and quadrilaterals.

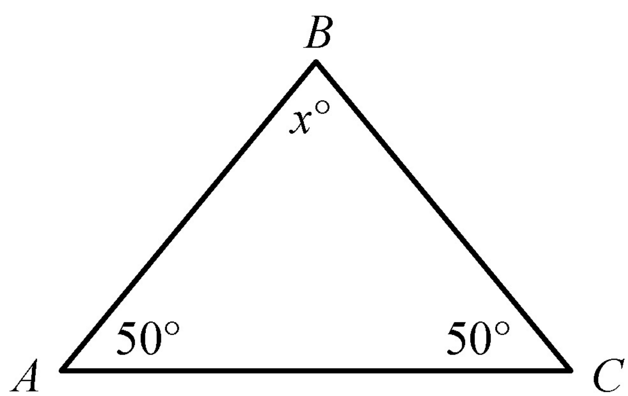
3.3 Triangles

Every triangle has three sides and three interior angles. The measures of the interior angles add up to 180°. The length of each side must be less than the sum of the lengths of the other two sides. For example, the sides of a triangle could not have the lengths 4, 7, and 12 because 12 is greater than $4 + 7$.

The following are 3 types of special triangles.

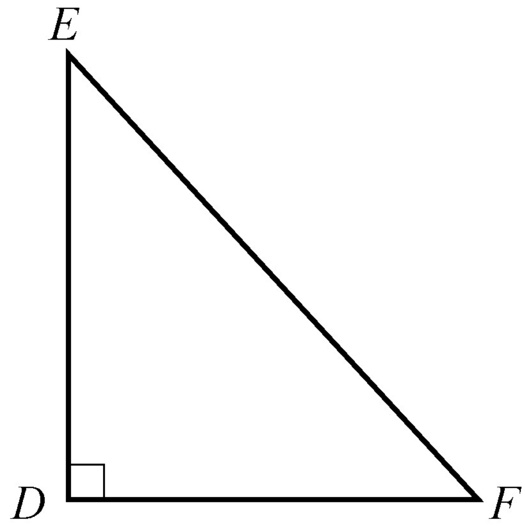
Type 1: A triangle with three congruent sides is called an **equilateral triangle**. The measures of the three interior angles of such a triangle are also equal, and each measure is 60°.

Type 2: A triangle with at least two congruent sides is called an **isosceles triangle**. If a triangle has two congruent sides, then the angles opposite the two sides are congruent. The converse is also true. For example, in $\triangle ABC$ in Geometry Figure 8 below, the measure of $\angle A$ is 50°, the measure of $\angle C$ is 50°, and the measure of angle B is x°. Since both $\angle A$ and $\angle C$ have measure 50°, it follows that $AB = BC$. Also, since the sum of the 3 angles of a triangle is 180°, it follows that $50 + 50 + x = 180$, and the measure of $\angle B$ is 80°.



Geometry Figure 8

Type 3: A triangle with an interior right angle is called a **right triangle**. The side opposite the right angle is called the **hypotenuse**; the other two sides are called **legs**.

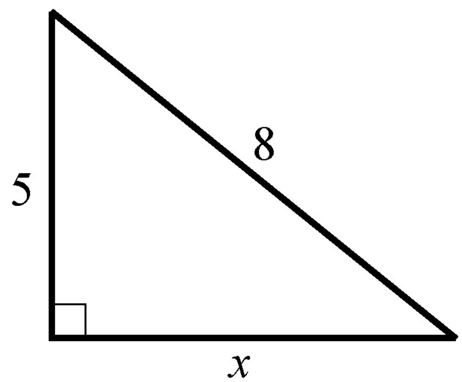


Geometry Figure 9

In right triangle *DEF* in Geometry Figure 9 above, side *EF* is the side opposite right angle *D*, therefore *EF* is the hypotenuse and *DE* and *DF* are legs. The **Pythagorean theorem** states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. Thus, for triangle *DEF* in Geometry Figure 9 above,

$$(EF)^2 = (DE)^2 + (DF)^2.$$

This relationship can be used to find the length of one side of a right triangle if the lengths of the other two sides are known. For example, consider a right triangle hypotenuse of length 8, a leg of length 5, and another leg of unknown length x, as shown in Geometry Figure 10 below.



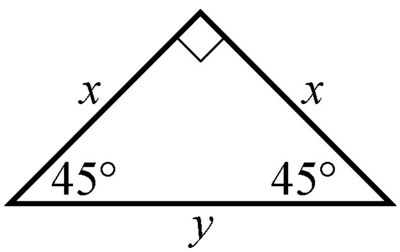
Geometry Figure 10

By the Pythagorean theorem, $8^2 = 5^2 + x^2$.

Therefore, $64 = 25 + x^2$ and $39 = x^2$.

Since $x^2 = 39$ and x must be positive, it follows that $x = \sqrt{39}$, or approximately 6.2.

The Pythagorean theorem can be used to determine the ratios of the sides of two special right triangles. One special right triangle is an isosceles right triangle, as shown in Geometry Figure 11 below.



Geometry Figure 11

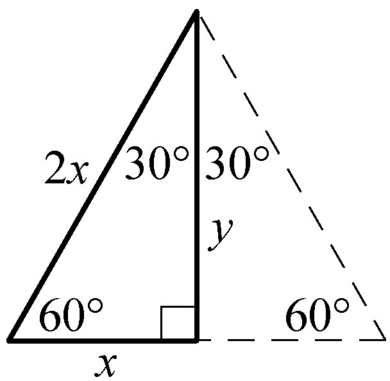
In Geometry Figure 11, the hypotenuse of the right triangle is of length *y,* both legs are of length *x*, and the angles opposite the legs are both 45 degree angles.

Applying the Pythagorean theorem to the isosceles right triangle in Geometry Figure 11 shows that the lengths of its sides are in the ratio 1 to 1 to $\sqrt{2}$, as follows.

By the Pythagorean theorem, $y^2 = x^2 + x^2$.

Therefore $y^2 = 2x^2$ and $y = \sqrt{2}x$. So the lengths of the sides are in the ratio x to x to $\sqrt{2}x$, which is the same as the ratio 1 to 1 to $\sqrt{2}$.

The other special right triangle is a $30° \text{-} 60° \text{-} 90°$ right triangle, which is half of an equilateral triangle, as shown in Geometry Figure 12 below.



Geometry Figure 12

Note that the length of the horizontal side, x, is one-half the length of the hypotenuse, $2x$. Applying the Pythagorean theorem to the $30° \text{-} 60° \text{-} 90°$ right triangle shows that the lengths of its sides are in the ratio 1 to $\sqrt{3}$ to 2, as follows.

By the Pythagorean theorem, $x^2 + y^2 = (2x)^2$, which simplifies to $x^2 + y^2 = 4x^2$.

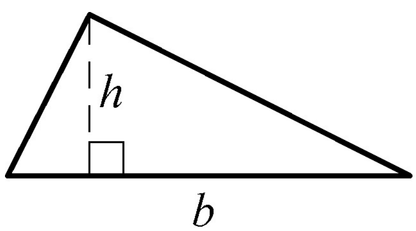
Subtracting x^2 from both sides gives $y^2 = 4x^2 - x^2$ or $y^2 = 3x^2$.

Therefore, $y = \sqrt{3}x$.

Hence, the ratio of the lengths of the three sides of a $30°$-$60°$-$90°$ right triangle is x to $\sqrt{3}x$ to $2x$, which is the same as the ratio 1 to $\sqrt{3}$ to 2.

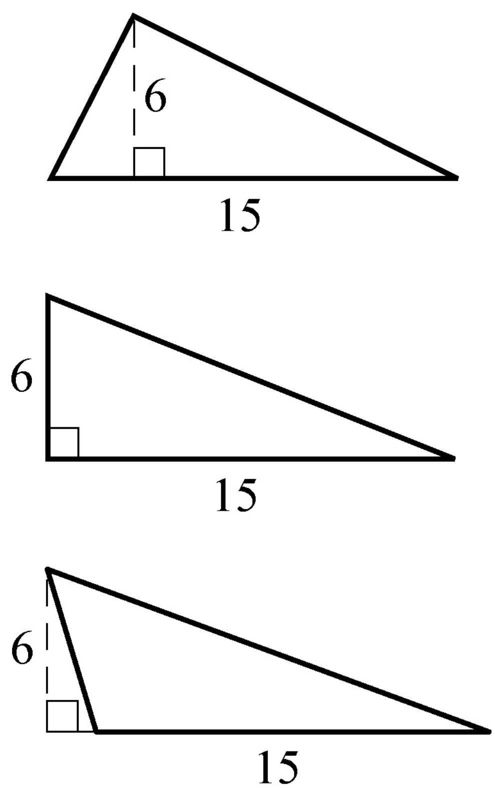
The **area** A of a triangle equals one-half the product of the length of a base and the height corresponding to the base, or $A = \frac{bh}{2}$.

In Geometry Figure 13 below, the horizontal base is denoted by b and the corresponding vertical height is denoted by h.



Geometry Figure 13

Any side of a triangle can be used as a base; the height that corresponds to the base is the perpendicular line segment from the opposite vertex to the base (or an extension of the base). The examples in Geometry Figure 14 below show three different configurations of a base and the corresponding height.



Geometry Figure 14

In all three triangles in Geometry Figure 14 above, the area is $\frac{(15)(6)}{2}$, or 45.

Two triangles that have the same shape and size are called **congruent triangles**. More precisely, two triangles are congruent if their vertices can be matched up so that the corresponding angles and the corresponding sides are congruent.

The following three propositions can be used to determine whether two triangles are congruent by comparing only some of their sides and angles.

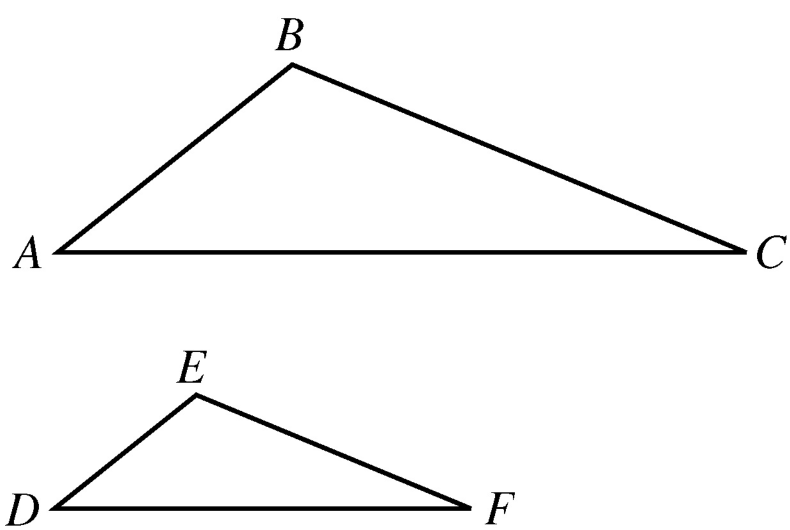
Proposition 1: If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

Proposition 2: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Proposition 3: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Two triangles that have the same shape but not necessarily the same size are called **similar triangles**. More precisely, two triangles are similar if their vertices can be matched up so that the corresponding angles are congruent or, equivalently, the lengths of corresponding sides have the same ratio, called the scale factor of similarity. For example, all $30°$-$60°$-$90°$ right triangles, are similar triangles, though they may differ in size.

When we say that triangles *ABC* and *DEF* are similar, it is assumed that angles *A* and *D* are congruent, angles *B* and *E* are congruent, and angles *C* and *F* are congruent, as shown in Geometry Figure 15 below. Also sides *AB*, *BC*, and *AC* in triangle *ABC* correspond to sides *DE*, *EF*, and *DF* in triangle *DEF*, respectively. In other words, the order of the letters indicates the correspondences.



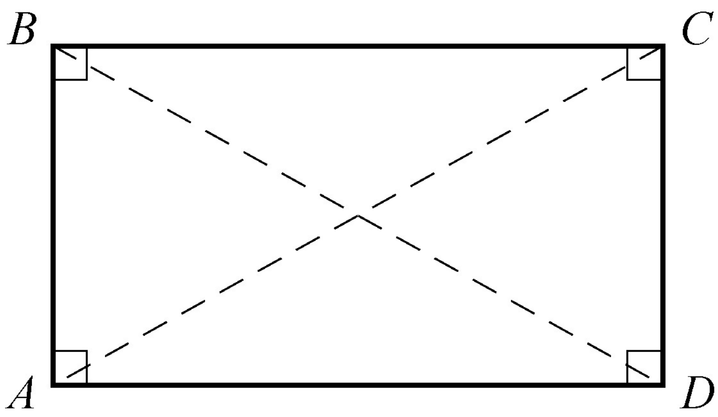
Geometry Figure 15

Since triangles ABC and DEF are similar, we have $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
By cross multiplication, we can obtain other proportions, such as
$\frac{AB}{BC} = \frac{DE}{EF}$.

3.4 Quadrilaterals

Every quadrilateral has four sides and four interior angles. The measures of the interior angles add up to $360°$. The following are four special types of quadrilaterals.

Type 1: A quadrilateral with four right angles is called a **rectangle**. Opposite sides of a rectangle are parallel and congruent, and the two diagonals are also congruent.



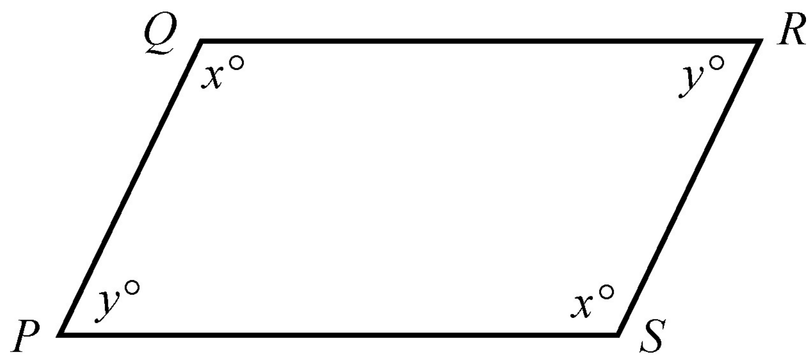
Geometry Figure 16

Geometry Figure 16 above shows rectangle *ABCD*.

In rectangle *ABCD*, opposite sides *AD* and *BC* are parallel and congruent, opposite sides *AB* and *DC* are parallel and congruent, and diagonal *AC* is congruent to diagonal *BD*.

Type 2: A rectangle with four congruent sides is called a **square**.

Type 3: A quadrilateral in which both pairs of opposite sides are parallel is called a **parallelogram**. In a parallelogram, opposite sides are congruent and opposite angles are congruent.



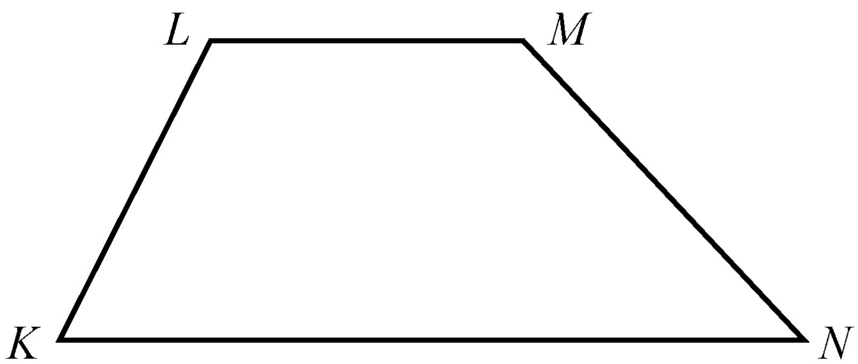
Geometry Figure 17

Geometry Figure 17 above shows parallelogram *PQRS*.

In parallelogram *PQRS*,
opposite sides *PQ* and *SR* are parallel and congruent,
opposite sides *QR* and *PS* are parallel and congruent
opposite angles *Q* and *S* are congruent, and
opposite angles *P* and *R* are congruent.

Note that in the figure angles *Q* and *S* are both labeled x°, and angles *P* and *R* are both labeled y°.

Type 4: A quadrilateral in which two opposite sides are parallel is called a **trapezoid**.



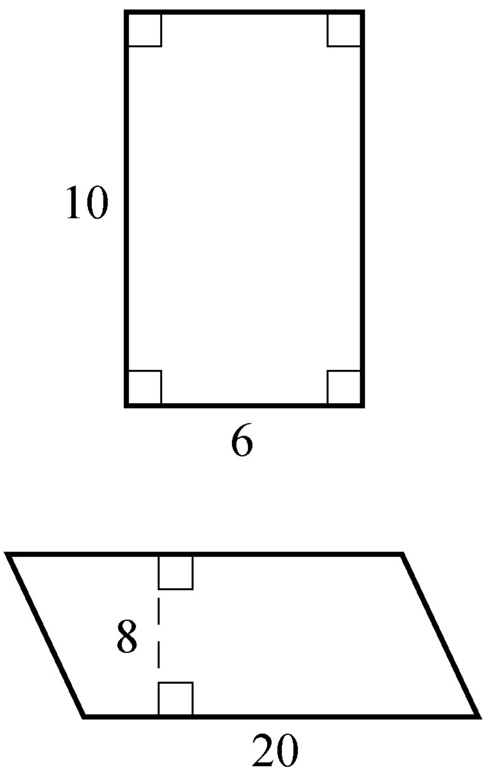
Geometry Figure 18

Geometry Figure 18 above shows trapezoid *KLMN*. In trapezoid *KLMN*, horizontal side *KN* is parallel to horizontal side *LM*.

For all parallelograms, including rectangles and squares, the **area** *A* equals the product of the length of a base *b* and the corresponding height *h*; that is,

$A = bh.$

Any side can be used as a base. The height corresponding to the base is the perpendicular line segment from any point of a base to the opposite side (or an extension of that side). In Geometry Figure 19 below are examples of finding the areas of a rectangle and a parallelogram.



$A = (6)(10) = 60$

$A = (20)(8) = 160$

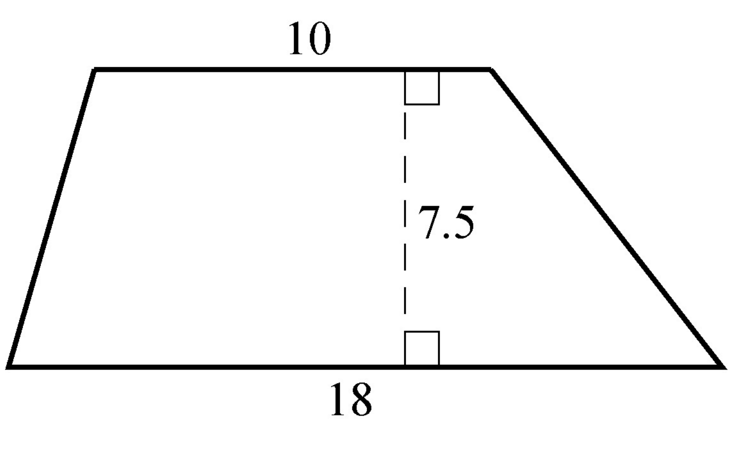
Geometry Figure 19

The **area** A of a trapezoid equals half the product of the sum of the lengths of the two parallel sides b_1 and b_2 and the corresponding height h; that is,

$$A = \frac{1}{2}(b_1 + b_2)h.$$

For example, for the trapezoid in Geometry Figure 20 below with bases of length 10 and 18 and a height of 7.5, the area is

$$A = \frac{1}{2}(10 + 18)(7.5) = 105.$$

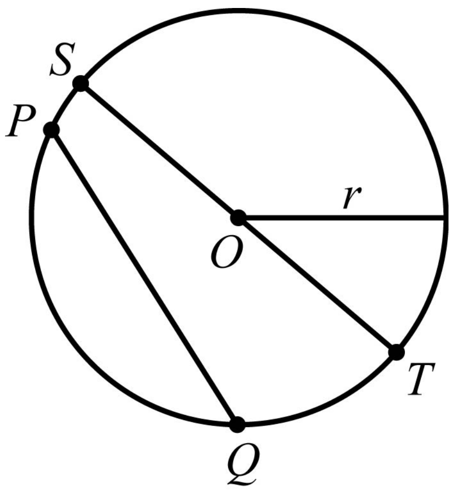


Geometry Figure 20

3.5 Circles

Given a point *O* in a plane and a positive number *r*, the set of points in the plane that are a distance of *r* units from *O* is called a **circle**. The point *O* is called the **center** of the circle and the distance *r* is called the **radius** of the circle. The **diameter** of the circle is twice the radius. Two circles with equal radii are called **congruent circles**.

Any line segment joining two points on the circle is called a **chord**. The terms "radius" and "diameter" can also refer to line segments: A **radius** is any line segment joining a point on the circle and the center of the circle, and a **diameter** is a chord that passes through the center of the circle. In Geometry Figure 21 below, O is the center of the circle, r is the radius, PQ is a chord, and ST is a diameter.



Geometry Figure 21

The distance around a circle is called the **circumference** of the circle, which is analogous to the perimeter of a polygon. The ratio of the circumference C to the diameter d is the same for all circles and is denoted by the Greek letter π; that is,

$$\frac{C}{d} = \pi.$$

The value of π is approximately 3.14 and can also be approximated by the fraction $\frac{22}{7}$. If r is the radius of a circle, then $\frac{C}{d} = \frac{C}{2r} = \pi$, and so the circumference is related to the radius as follows.

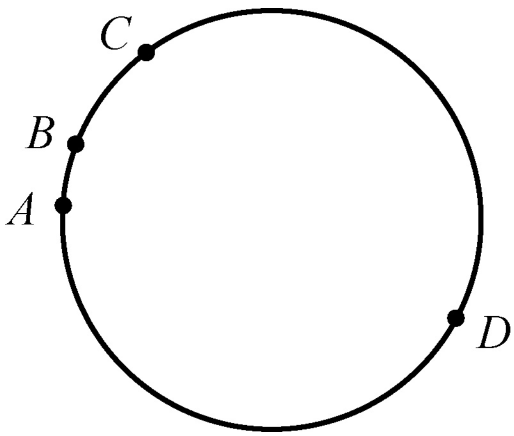
$$C = 2\pi r$$

For example, if a circle has a radius of 5.2, then its circumference is

$$(2)(\pi)(5.2) = (10.4)(\pi) \approx (10.4)(3.14),$$

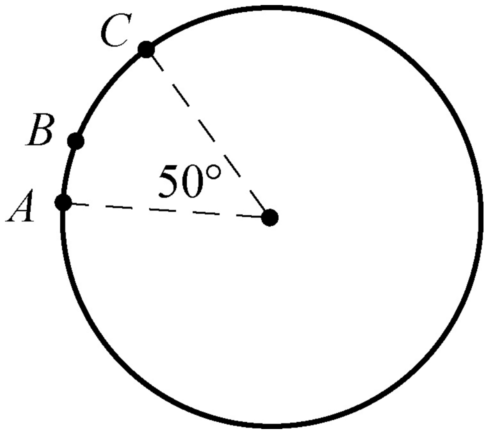
which is approximately 32.7.

Given any two points on a circle, an **arc** is the part of the circle containing the two points and all the points between them. Two points on a circle are always the endpoints of two arcs. It is customary to identify an arc by three points to avoid ambiguity. In Geometry Figure 22 below, there are four points on a circle. Going clockwise around the circle the four points are *A*, *B*, *C*, and *D*. There are two different arcs between points *A* and *C*: arc *ABC* is the shorter arc between *A* and *C*, and arc *ADC* is the longer arc between *A* and *C*.



Geometry Figure 22

A **central angle** of a circle is an angle with its vertex at the center of the circle. The **measure of an arc** is the measure of its central angle, which is the angle formed by two radii that connect the center of the circle to the two endpoints of the arc. An entire circle is considered to be an arc with measure 360°.



Geometry Figure 23

In Geometry Figure 23, the measure of the shorter arc between points *A* and *C*, that is arc *ABC*, is $50°$; and the measure of the longer arc between points *A* and *C* is $310°$.

In addition to the information given in the figure, it is also given that the radius of the circle is 5.

To find the **<u>length of an arc</u>** of a circle, note that the ratio of the length of an arc to the circumference is equal to the ratio of the degree measure of the arc to $360°$. For example, since the radius of the circle in Geometry Figure 23 is 5, the circumference of the circle is 10π.

Therefore,

$$\frac{\text{length of arc } ABC}{10\pi} = \frac{50}{360}$$

$$\text{length of arc } ABC = \left(\frac{50}{360}\right)(10\pi) = \frac{25\pi}{18} \approx \frac{(25)(3.14)}{18} \approx 4.4.$$

The **area** of a circle with radius r is equal to πr^2. For example, since the circle in Geometry Figure 23 has radius 5, the area of the circle is $\pi(5)^2 = 25\pi$.

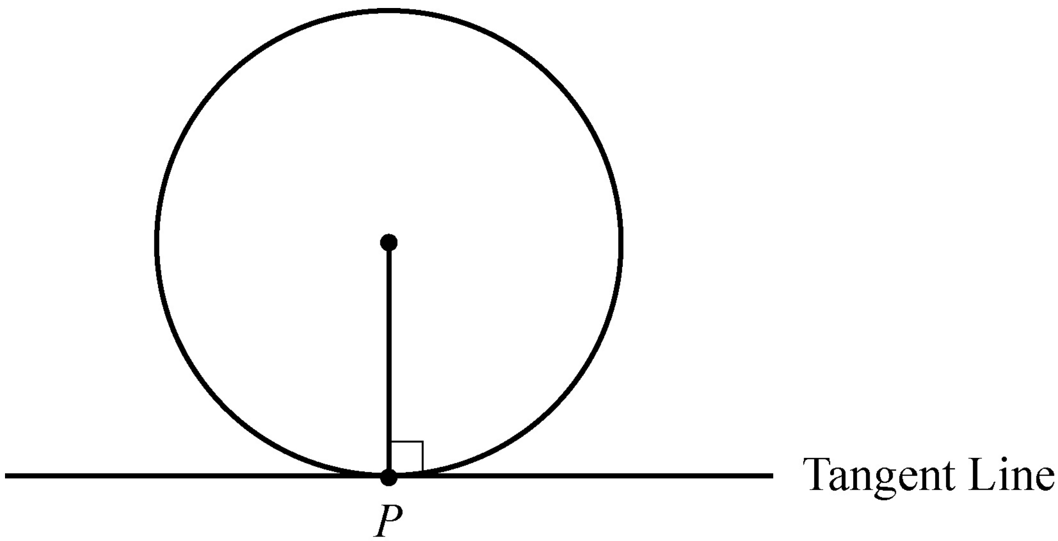
A **sector** of a circle is a region bounded by an arc of the circle and two radii. In the circle in Geometry Figure 23 above, the region bounded by arc ABC and the two radii is a sector with central angle $50°$. Just as in the case of the length of an arc, the ratio of the area of a sector of a circle to the area of the entire circle is equal to the ratio of the degree measure of its arc to $360°$. Therefore, if S represents the area of the sector with central angle $50°$, then

$$\frac{S}{25\pi} = \frac{50}{360}.$$

To solve for S, multiply both sides by 25π and then simplify as follows.

$$S = \left(\frac{50}{360}\right)(25\pi) = \frac{125\pi}{36} \approx \frac{(125)(3.14)}{36} \approx 10.9.$$

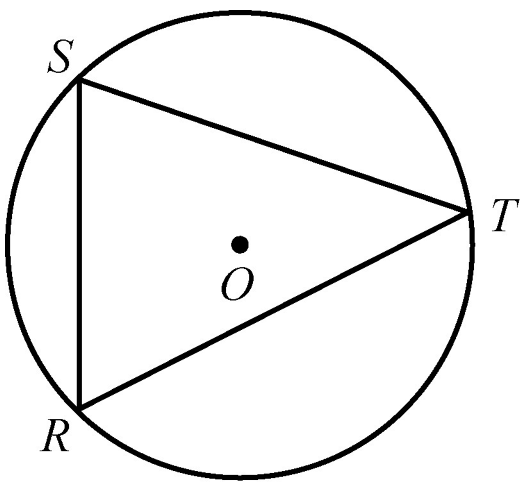
A **tangent** to a circle is a line that intersects the circle at exactly one point, called the **point of tangency**. If a line is tangent to a circle, then a radius drawn to the point of tangency is perpendicular to the tangent line. Geometry Figure 24 below shows a circle, a line tangent to the circle at point P, and a radius drawn to point P. The converse is also true; that is, if a line is perpendicular to a radius at its endpoint on the circle, then the line is a tangent to the circle at that endpoint.



Geometry Figure 24

A polygon is **inscribed** in a circle if all its vertices lie on the circle, or equivalently, the circle is **circumscribed** about the polygon.

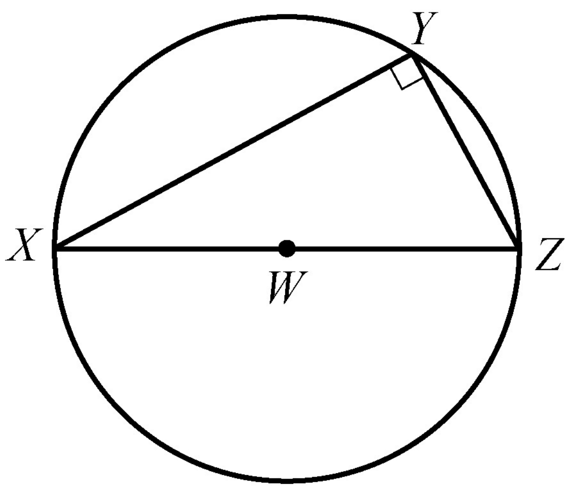
Geometry Figure 25 below shows triangle *RST* inscribed in a circle with center *O*. The center of the circle is inside the triangle.



Geometry Figure 25

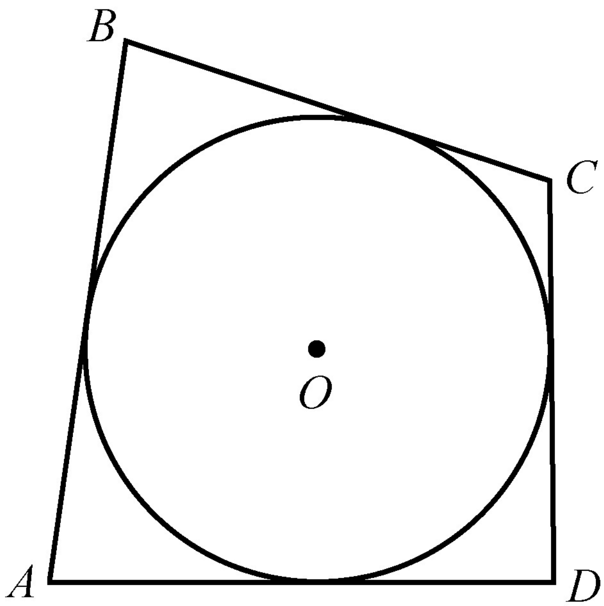
If one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle. Conversely, if an inscribed triangle is a right triangle, then one of its sides is a diameter of the circle.

Geometry Figure 26 below shows right triangle *XYZ* inscribed in a circle with center *W*. In triangle *XYZ*, side *XZ* is a diameter of the circle and angle *Y* is a right angle.



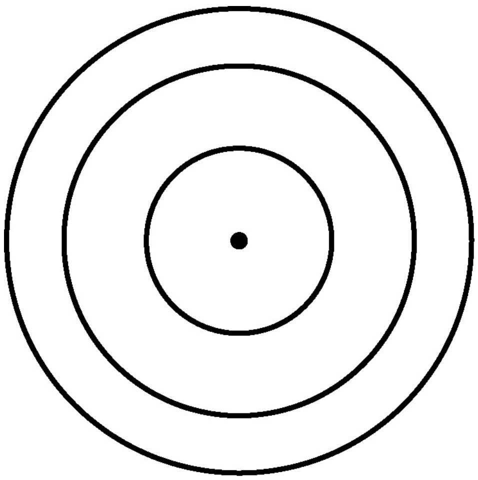
Geometry Figure 26

A polygon is circumscribed about a circle if each side of the polygon is tangent to the circle, or equivalently, the circle is inscribed in the polygon. Geometry Figure 27 below shows quadrilateral *ABCD* circumscribed about a circle with center *O*.



Geometry Figure 27

Two or more circles with the same center are called **concentric circles**, as shown in Geometry Figure 28 below.

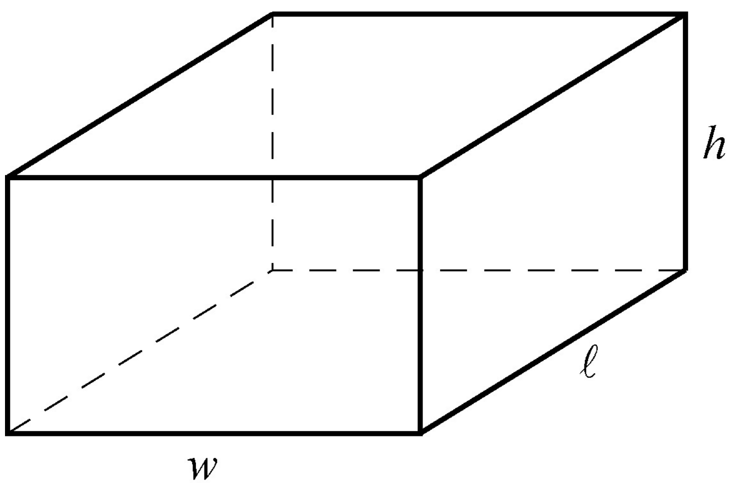


Geometry Figure 28

3.6 Three-Dimensional Figures

Basic three-dimensional figures include rectangular solids, cubes, cylinders, spheres, pyramids, and cones. In this section, we look at some properties of rectangular solids and right circular cylinders.

A **rectangular solid** has six rectangular surfaces called **faces**, as shown in Geometry Figure 29 below. Adjacent faces are perpendicular to each other. Each line segment that is the intersection of two faces is called an **edge**, and each point at which the edges intersect is called a **vertex**. There are 12 edges and 8 vertices. The dimensions of a rectangular solid are the length ℓ, the width w, and the height h.



Geometry Figure 29

A rectangular solid with six square faces is called a **cube**, in which case $\ell = w = h$.

The **volume** V of a rectangular solid is the product of its three dimensions, or

$$V = \ell w h.$$

The **surface area** A of a rectangular solid is the sum of the areas of the six faces, or

$$A = 2(\ell w + \ell h + wh).$$

For example, if a rectangular solid has length 8.5, width 5, and height 10, then its volume is

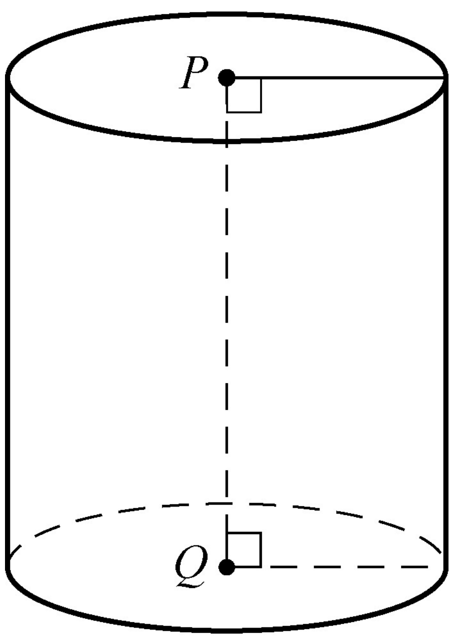
$$V = (8.5)(5)(10) = 425$$

and its surface area is

$$A = 2((8.5)(5) + (8.5)(10) + (5)(10)) = 355.$$

A **circular cylinder** consists of two bases that are congruent circles and a **lateral surface** made of all line segments that join points on the two circles and that are parallel to the line segment joining the centers of the two circles. The latter line segment is called the **axis** of the cylinder. A **right circular cylinder** is a circular cylinder whose axis is perpendicular to its bases.

The right circular cylinder shown in Geometry Figure 30 below has circular bases with centers P and Q. Line segment PQ is the axis of the cylinder and is perpendicular to both bases. The length of PQ is called the height of the cylinder.



Geometry Figure 30

The **volume** V of a right circular cylinder that has height h and a base with radius r is the product of the height and the area of the base, or

$$V = \pi r^2 h.$$

The **surface area** A of a right circular cylinder is the sum of the areas of the two bases and the lateral area, or

$$A = 2\left(\pi r^2\right) + 2\pi rh.$$

For example, if a right circular cylinder has height 6.5 and a base with radius 3, then its volume is

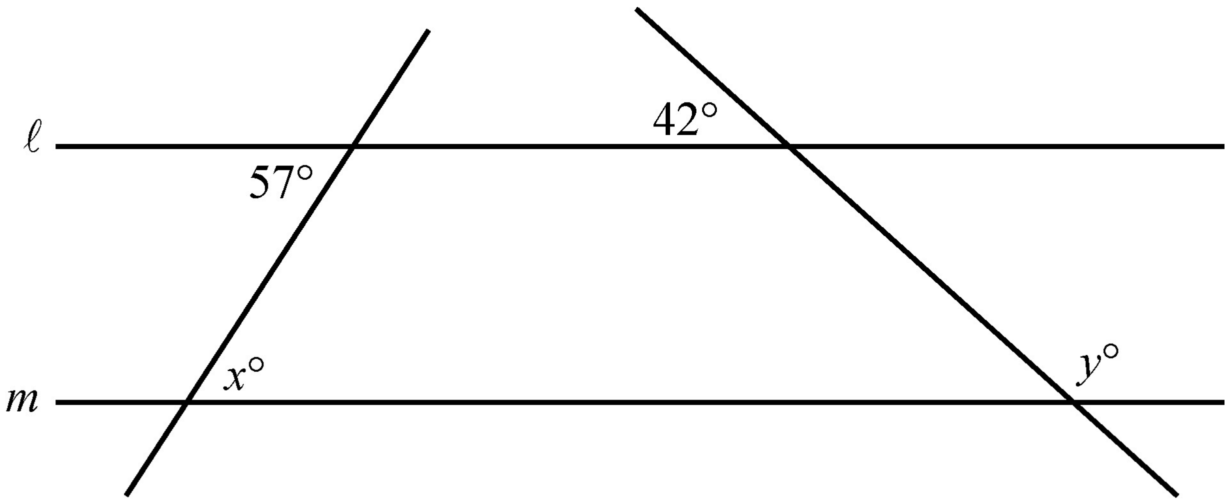
$$V = \pi(3)^2(6.5) = 58.5\pi$$

and its surface area is

$$A = (2)(\pi)(3)^2 + (2)(\pi)(3)(6.5) = 57\pi.$$

Geometry Exercises

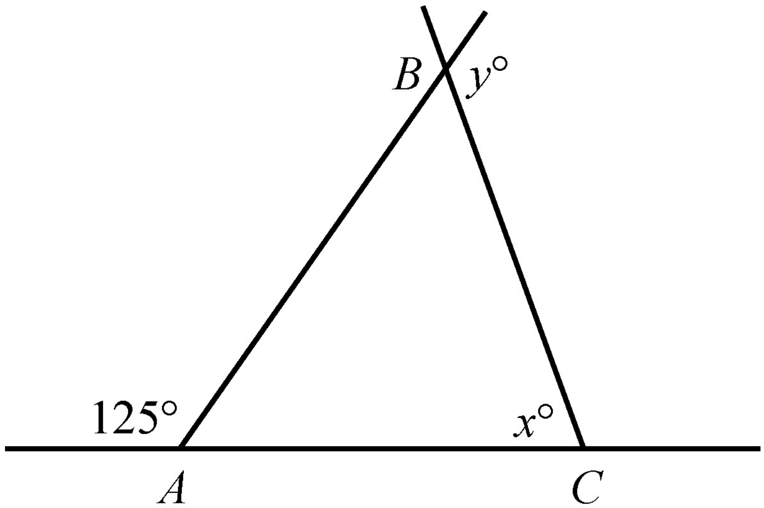
1. Exercise 1 is based on Geometry Figure 31 below.



Geometry Figure 31

In Geometry Figure 31, lines ℓ and m are parallel. Find the values of x and y.

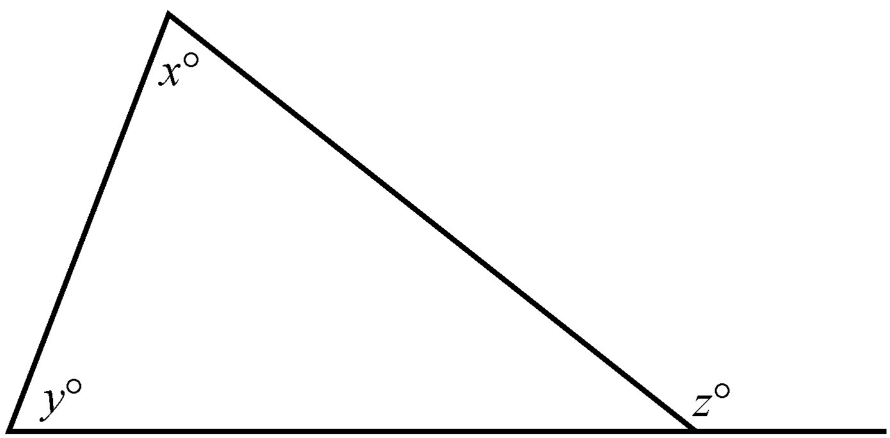
2. Exercise 2 is based on Geometry Figure 32 below.



Geometry Figure 32

In Geometry Figure 32, $AC = BC$. Find the values of x and y.

3. Exercise 3 is based on Geometry Figure 33 below.



Geometry Figure 33

In Geometry Figure 33, what is the relationship between x, y, and z ?

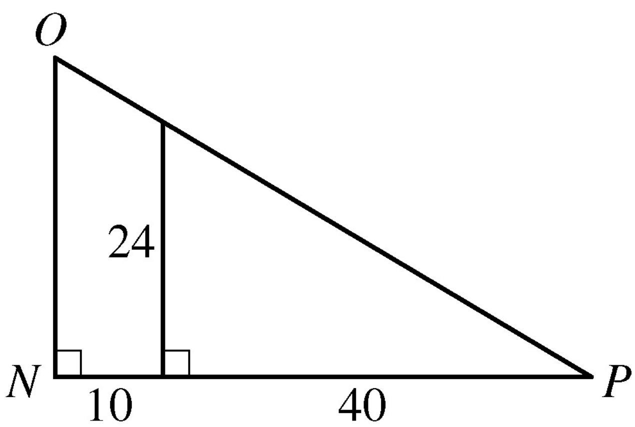
4. What is the sum of the measures of the interior angles of a decagon (10-sided polygon) ?

5. If the polygon in exercise 4 is regular, what is the measure of each interior angle?

6. The lengths of two sides of an isosceles triangle are 15 and 22, respectively. What are the possible values of the perimeter?

7. Triangles PQR and XYZ are similar. If $PQ = 6$, $PR = 4$, and $XY = 9$, what is the length of side XZ? (Note that there is no figure accompanying this exercise.)

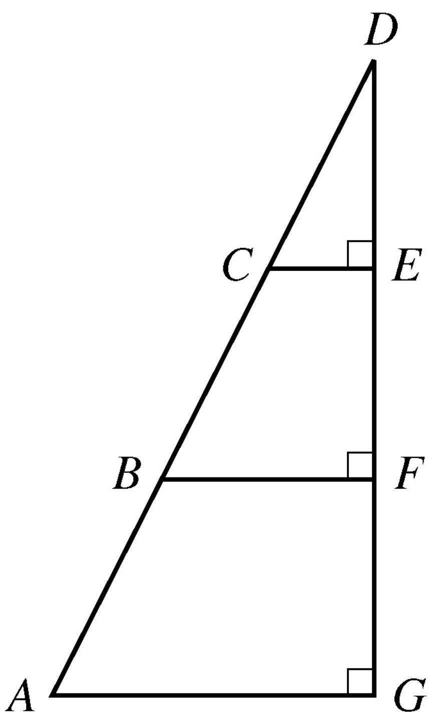
8. Exercise 8 is based on Geometry Figure 34 below.



Geometry Figure 34

In Geometry Figure 34, what are the lengths of sides *NO* and *OP* in triangle *NOP* ?

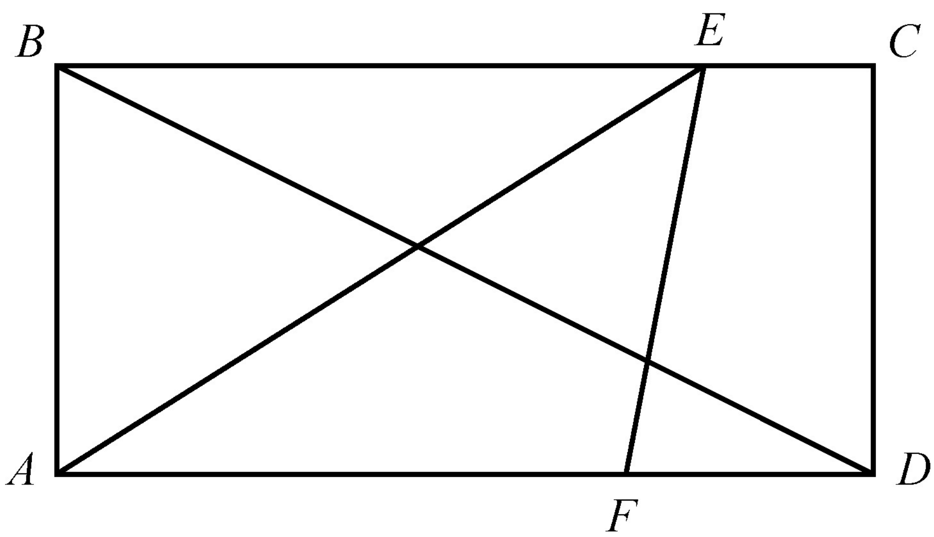
9. Exercise 9 is based on Geometry Figure 35 below.



Geometry Figure 35

In Geometry Figure 35, $AB = BC = CD$. If the area of triangle CDE is 42, what is the area of triangle ADG ?

10. Exercise 10 is based on Geometry Figure 36 below.



Geometry Figure 36

In Geometry Figure 36, *ABCD* is a rectangle, $AB = 5$, $AF = 7$, and $FD = 3$. Find the following.

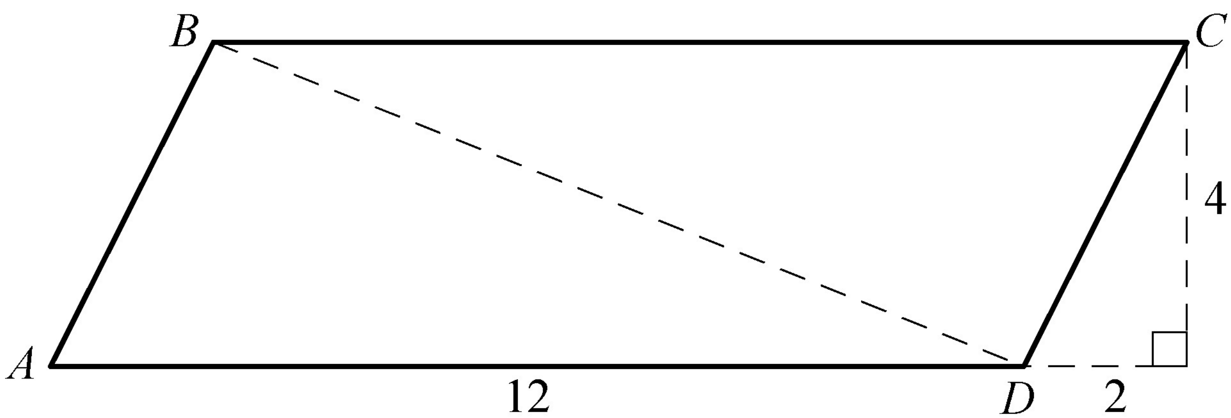
(a) Area of rectangle *ABCD*

(b) Area of triangle *AEF*

(c) Length of *BD*

(d) Perimeter of rectangle *ABCD*

11. Exercise 11 is based on Geometry Figure 37 below.



Geometry Figure 37

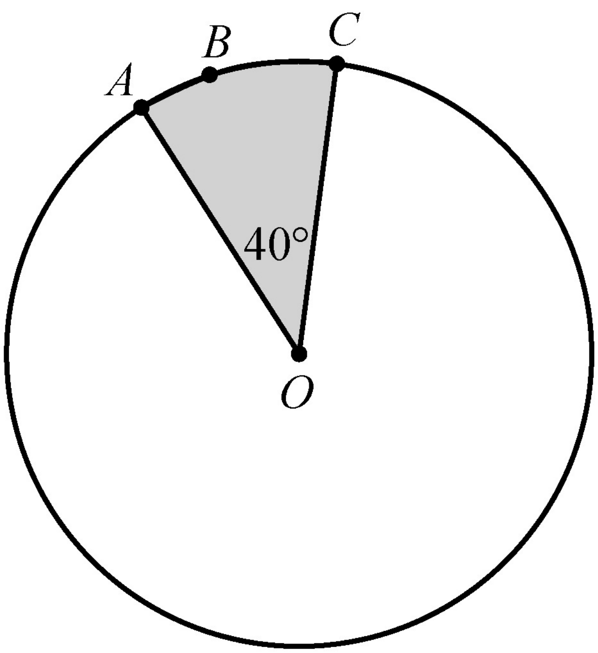
In parallelogram *ABCD* in Geometry Figure 37, find the following.

(a) Area of *ABCD*

(b) Perimeter of *ABCD*

(c) Length of diagonal *BD*

12. Exercise 12 is based on Geometry Figure 38 below.



Geometry Figure 38

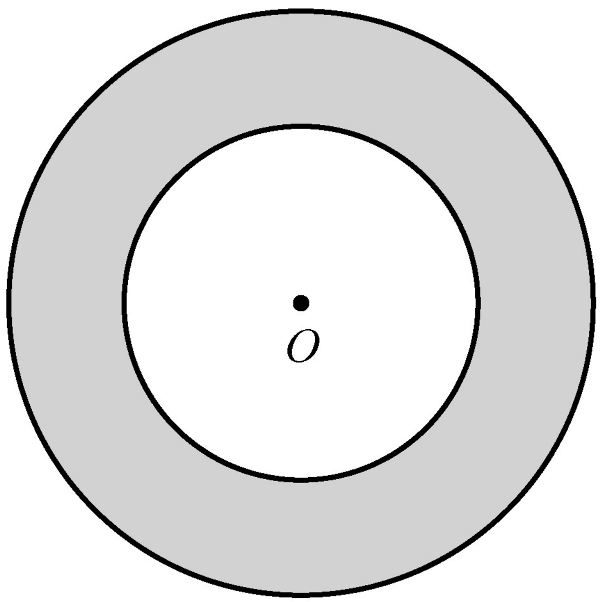
In Geometry Figure 38, the circle with center O has radius 4. Find the following.

(a) Circumference of the circle

(b) Length of arc ABC

(c) Area of the shaded region

13. Exercise 13 is based on Geometry Figure 39.



Geometry Figure 39

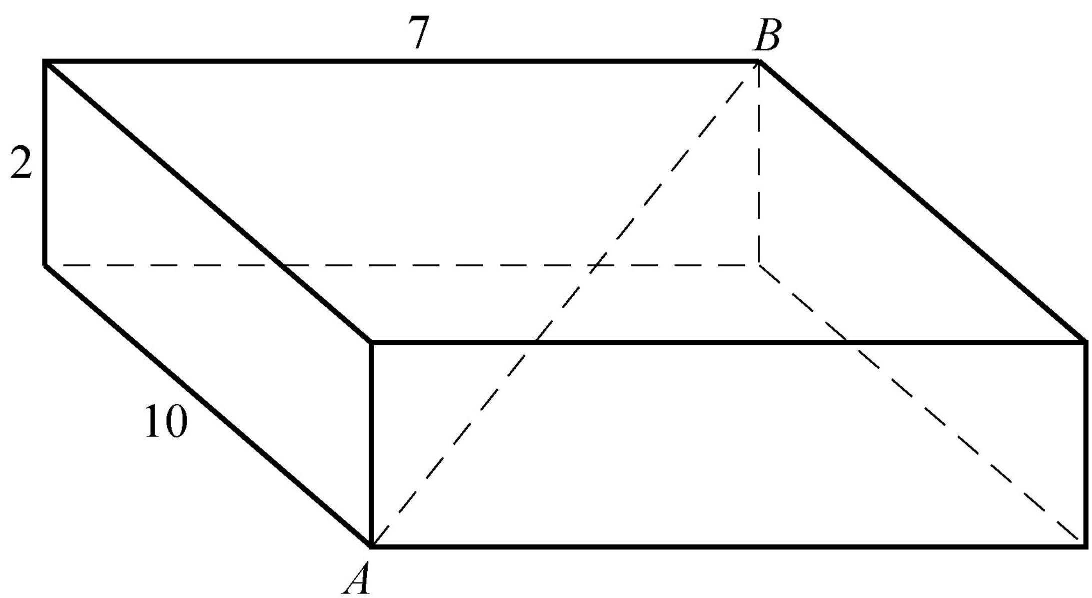
Geometry Figure 39 shows two concentric circles, each with center O. Given that the larger circle has radius 12 and the smaller circle has radius 7, find the following.

(a) Circumference of the larger circle

(b) Area of the smaller circle

(c) Area of the shaded region

14. Exercise 14 is based on Geometry Figure 40, which is a rectangular solid.



Geometry Figure 40

For the rectangular solid in Geometry Figure 40, find the following.

(a) Surface area of the solid

(b) Length of diagonal *AB*

Answers to Geometry Exercises

1. $x = 57$ and $y = 138$

2. $x = 70$ and $y = 125$

3. $z = x + y$

4. 1,440°

5. 144°

6. 52 and 59

7. 6

8. $NO = 30$ and $OP = 10\sqrt{34}$

9. 378

10. (a) 50

    (b) 17.5

    (c) $5\sqrt{5}$

    (d) 30

11. (a) 48

    (b) $24 + 4\sqrt{5}$

    (c) $2\sqrt{29}$

12. (a) 8π

    (b) $\frac{8\pi}{9}$

    (c) $\frac{16\pi}{9}$

13. (a) 24π

    (b) 49π

    (c) 95π

14. (a) 208

    (b) $3\sqrt{17}$